CHAPTER 5

Retentivity

As was seen in Chapter 4, the attrition rate and amount of attrition differs considerably among the countries represented in this project. In general, the USA and Japan have highly retentive systems of education in the sense that a high proportion of each year group continues through to the end of secondary education. In Europe, on the other hand, there is a much smaller proportion of a year group proceeding to the pre-university year. The different proportions are connected with the different philosophies of comprehensive and selective school systems as well as reflecting differing socio-economic structures between the countries. Secondary education in most European countries has been characterised, until recently, by the selection and transfer of "more able" pupils into separate types of academic school while the rest of the pupils have remained in schools initially designed to provide a basic education for the majority of children (e.g. *elementary scool, Volksschule, école primaire*).

The academic secondary school, with a long tradition going back to the medieval Latin school, has tended to recruit (select) the bulk of its pupils from the higher socio-economic strata. On the other hand, the development of public education in most parts of the United States has not been markedly affected by traditional practices, with the result that the eight year elementary schools were not regarded primarily as a preparation for secondary schooling, but as self-contained establishments capable of extending their provision to satisfy the educational needs of the community. Thus, in the European school systems, there developed the practice of selecting an élite to go through to the pre-university year, whereas in the more comprehensive systems (e.g. U.S.A.) the type of system was such that there grew up a deliberate policy of encouraging as many pupils as possible to continue through to the pre-university year (cf. Husén, 1962).

However, many of the European countries are at present revising their policies. Economic growth and the recent rapid advances in science and technology have created the need for a more prolonged period of general education for all young people and not just for the most able minority, with the result that successive increases in the duration of compulsory schooling have been made in most European countries. Furthermore, the need for more skilled and better informed manpower has also resulted in a substantial increase, in most countries, in the numbers of young people choosing to continue their education beyond the statutory school-leaving age. In Sweden, for example, in 1950 only ten percent of seventeen-year-olds proceeded to *gymnasiet*, while by 1964 the proportion had risen to twenty-eight percent (Yates, 1966). By 1970, it is estimated that nearly 30 percent will wish to enrol in *gymnasiet* (Dahllöf *et al.*, 1966). This increased proportion of a year group continuing to the end of secondary education is often accompanied by a restructuring of the educational system itself, either by the introduction of a comprehensive system of education with no selection or by delaying selection into the academic secondary school.

In the Case Study Questionnaire, data were collected on the actual number of students in each year group still in full-time schooling, as well as the actual number of students in each grade group. The national statistics which were the sources of these data were, in general, available, depending on the country, for the years between 1960 and 1963. In every case, it was the most recently available statistics which were used. Furthermore, the heads of National Centres were asked to estimate for 1964, at the time of testing, (a) the percentage of an age group in school at the pre-university level and (b) the proportion who were specialising in mathematics (enrolled in the terminal Mathematics-Science programmes). The division into mathematics and non-mathematics students in the pre-university year has already been discussed in Chapter 2. It would seem that in some National Centres approximations were made to the nearest whole number, whereas in others, the proportion was calculated to the first decimal place. The actual figures supplied are used in this analysis.

These figures are given in Table 5.1 in which there are also given, in the fourth column, measures of the degree to which each country has adopted a comprehensive system of education. This has been

assessed by the percentage of students in the younger and complete age group (Population 1a) attending so-called "comprehensive" schools. This information was collected by means of the School Questionnaire (see Appendix II, Volume I of Husén *et al.*, 1967). A comprehensive school was described as offering appropriate courses for students of *all* ranges of ability.

From Table 5.1 it can be clearly seen that there is considerable variation among the countries in this study in the percentage of a year group continuing through to the pre-university year. Since it has been possible to measure the mathematics achievement of the pre-university students as well as the 13-year-old students[1] in the countries, it is worthwhile posing several questions concerning the amount of mathematical achievement of both the pre-university groups (in terms of the percentage of a year group still in school) and the 13-year-old group of students.

Table 5.1. *Indices of retentivity and comprehensive education.*

Country	Retentivity (percentages of age group)			Comprehensiveness (percentages of Pop. 1a)
	Total	3a	3b	
Australia	23	14	9	70
Belgium	13	4	9	0
England	12	5	7	9
Fed. Rep. of Germany	11	4.7	6.5	0
Finland	14	7	7	0
France	11	5	—	0
Israel	—	7	—	96
Japan	57	8	49	100
Netherlands	8	5	3	0
Scotland	18	5.4	12.6	44
Sweden	23	16	7	64
U.S.A.	70	18	52	92

The rank correlations of the three indices of retentivity with the extent to which pupils are being educated in comprehensive schools are 0.89, 0.76, and 0.73 respectively.

[1] For descriptions of the pre-university populations see pp. 237–239 of Vol. I of Husén *et al.* (1967). For description of the 13-year-old grade group see p. 29 in this book.

First of all, it is possible to examine whether there is a difference in the average score of students (in both of the two pre-university populations) in systems with different amounts of retentivity, i.e., *if more students are allowed through will this lower the average standard of performance?* Secondly, it is possible to examine the relative performances of the students by certain international standards by taking the number of students above the 95th international percentile and then discovering, for each nation, (a) what percentage this is of the students in full-time schooling and (b) what percentage these students are of a year group. This analysis will assist in an examination of the problem of *whether or not the standard of performance of the best students in the pre-university year deteriorates if a larger percentage of an age group goes through to the pre-university year.* Thirdly, it is possible to *examine the mathematics performance "yield"* of the target populations in the study. By "yield" is meant *how many* students are brought *how far* (in this case in terms of mathematics achievement as measured by the IEA tests), within the framework of full-time schooling in the educational system. This takes into account both the number of persons (in terms of the percentage of an age group reaching a particular level) and the level of achievement per person, and is therefore not simply a comparison of means between countries, irrespective of the differing percentages of an age group making up the population being compared. In this last case, it is also possible to compare increase in "yield" between the 13-year-old age group (where virtually one hundred percent of an age group are still in school) and the pre-university group of students. Thus there are three main problems, all of which are related to retentivity, which will be examined: Average performance, Fixed international standards performance and Yield.

In this connection it should be pointed out that there are differences on some major independent variables among the pre-university populations in the countries participating in this project. There is a wide variation in the socio-economic status composition of this group, ranging from a composition somewhat similar to the general population in the U.S.A., to a predominantly middle-class composition in Germany. A second major disparity is the mean age[2] which ranges from 17 years 2 months in Australia to 19 years 10 months in the Federal Republic of Germany. A third variation lies in the average number of subjects studied in the pre-university year, ranging from three in England to nine or more in Belgium, France, Japan and the Netherlands. These discrepancies have been dealt with to some extent in Chapter 4 of this book and in much more detail in Chapter 2 of Volume I of the IEA publication (Husén *et al.*, 1967).

[2] For a different analysis of age, retentivity and score see p. 68 *et seq.*, in Husén *et al.* Vol. II (1967).

In the discussion of yield, Population 1b has been used rather than Population 1a, although the latter would have been better since it is a chronologically comparable group. However, four countries (Australia, France, Israel and the Netherlands) were lost at the pre-university level, since either they did not test Population 3b, or their sampling procedures were considered to be inadequate. If 1a had been chosen for the lower level rather than 1b, there would have only been seven countries left, since Germany did not test 1a. Hence, Population 1b was chosen.

Average Performance

The percentages of an age group still in school (circa 1964) in the two pre-university populations have been given in Table 5.1. The

Table 5.2. *Total mathematics score, means, standard deviations and N's for populations 3a and 3b.*

	Pre-university math-science programme Population 3a			Pre-university non math-science programme Population 3b		
Country	M	S.D.	N	M	S.D.	N
Australia	21.6	10.5	1089	—	—	—
Belgium	34.6	12.6	519	24.2	9.5	1004
England	35.2	12.6	967	21.4	10.0	1782
Fed. Rep. of Germany	28.8	9.8	649	27.7	7.6	643
Finland	25.3	9.6	369	22.5	8.3	399
France	33.4	10.8	222	—	—	—
Israel	36.4	8.6	146	—	—	—
Japan	31.4	14.8	818	25.3	14.3	4372
Netherlands	31.9	8.1	462	—	—	—
Scotland	25.5	10.4	1422	20.7	9.5	2123
Sweden	27.3	11.9	776	12.6	6.2	222
U.S.A.	13.8	12.6	1568	8.3	9.0	2042

means, standard deviations and N's for Populations 3a and 3b are given in Table 5.2.

The relation of mathematics score to the percentage of an age group in school by country is shown for Populations 3a and 3b in Figures 5.1 and 5.2 respectively. The rank correlations between the mean score and the percentages of an age group in school in each population are −.62 and −.36 for Populations 3a and 3b respectively. The decrease in mean score as the percentage of an age group retained in school increases is clearly discernible in both populations, giving weight to the contention that the greater the retentivity, the lower will be the average score of those retained. It might also be thought that the smaller the percentage of an age group retained,

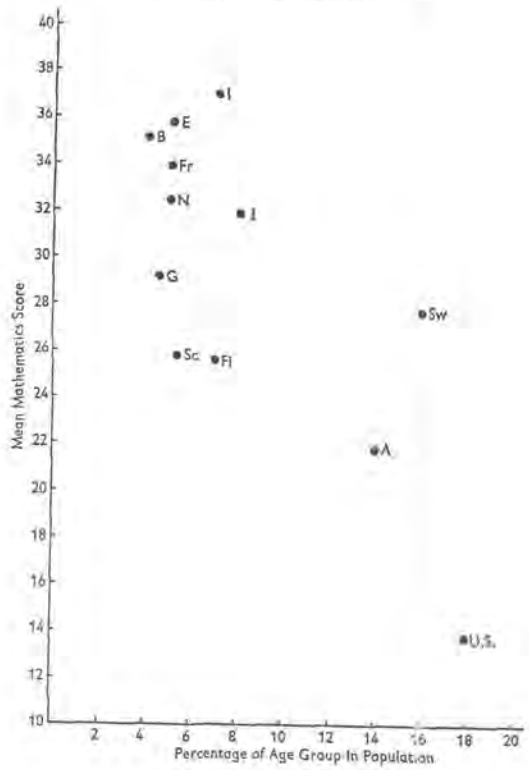


Fig. 5.1. *Relation of Mean Mathematics Score to Percentage of Age Group in Population by Country*

(Population 3 a)

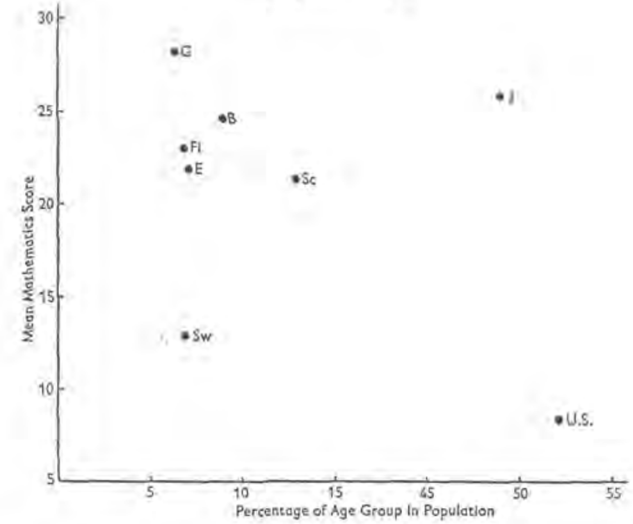


Fig. 5.2. *Relation of Mean Mathematics Score to Percentage of Age Group in Population by Country*

(Population 3 b)

the smaller would be the standard deviation, since those retained are likely to be more homogeneous in terms of mathematics achievement. There is some support for this, since the rank correlations between the percentage of an age group in school and standard deviation are .20 and .60 for Populations 3a and 3b respectively. The standard deviation is more likely to depend on how the groups retained are organized either within schools or between schools, and not just on the proportion retained. This must be a matter for further research.

Fixed International Standards Performance

Apart from examining the relationship between average scores and retentivity between countries, it is also interesting to employ another method of examining this problem — that of fixing a set of international standards to find what proportion of its pre-university students each country has been able to bring to each of these standards. Thus, we can examine not only what is achieved by the best students in each country, but also by the less able.

It has already been pointed out that there are major variations among the pre-university populations in the various countries in terms of some independent variables. With all these differences in mind, one might query whether it is justifiable to use combined distributions of scores from all countries as a base from which to derive percentiles for international comparisons. The reply would be that, whatever the national populations that contributed to produce them, the scores marked by the 95th and 85th percentiles of the combined distributions denote fixed points which can be used for at least some comparisons. For example, the 95th percentile for Population 3a is the score exceeded by only the best five percent of the combined pre-university populations for that level. If this five percent were composed of exactly five percent from each of the national pre-university populations, we should conclude that, in this respect at least, all the participating countries were equal. If the five percent international élite is not so composed, the question arises whether the differences are attributable, in part at least, to the varying percentages of the age group still at school.

Table 5.3 presents for each country the percentage of those students in Population 3a reaching six different *international* percentile levels.

Table 5.3. *Percentage of pre-university mathematics students reaching given standards.*

Country	Retentivity	International percentiles					
		25th	50th	75th	85th	90th	95th
	(1)	(2)	(3)	(4)	(5)	(6)	(7)
U.S.A.	18	36	18	9	7	4.5	3.6
Sweden	16	81	53	26	13	8	3.1
Australia	14	67	37	10	5	3	1.1
Japan	8	82	63	43	29.4	21	10.0
Finland	7	81	48	18	6	3.4	1.2
Scotland	5.4	83	44	16	9	6	3.7
England	5	94	79	50	34	26	12.0
France	5	92	69	39	29.2	22	9.0
Netherlands	5	97	77	35	14	5	1.3
Fed. Rep. of Germany	4.7	90	63	26	11	7	2.0
Belgium	4	90	70	44	30	23	21.0
Range		61	61	41	29	23	19.9
Rank correlation with column 1		−.61	−.72	−.47	−.59	−.52	−.35

For example, 36 percent of the 3a Population in the U.S.A. reached the 25th percentile level, as compared with 97 percent of the 3a Population in the Netherlands. First decimal places have been added to some entries to increase the precision of the rank correlations. These rank order correlations between the percentage of an age group in Population 3a (i.e. Column 1 in Table 5.3) and the percentage of that population reaching each percentile level are shown in the last row of the table.

The negative correlations indicate that the smaller the proportion of the total age group taking the mathematics programme at the pre-university stage, the larger will be the proportions reaching given levels of performance. Thus, those who maintain that increasing the intake will lower the "standards" have a point, particularly in terms of the bottom half of those taken in. However, it is of interest that the effect at the upper end of the distribution is weaker. The between country ranges of percentages scoring above various international percentile points are very large, ranging from 61 percent at the 25th and 50th percentiles to 19.9 percent at the 95th percentile (see Table 5.3). Of those countries where only four or five percent of an age group are enrolled in the mathematics programme, Belgium and England are outstanding, particularly in the top international quartile. It is remarkable that 21 percent of Belgian students achieve scores above the 95th percentile (as, for example, compared with 12 percent in England) when it is remembered that Belgian students are studying an average of six more subjects than English students. The Netherlands, on the other hand, has a high proportion of students up to the 50th international percentile, but a rapid fall then occurs. The U.S.A. is consistently lower than Sweden (except at the 95th percentile), whereas Japan is consistently higher than Scotland (except at the 25th percentile).

If there were no relation between the degree of retention and the scores made by the students retained, we might expect that each country would have 5% of their 3a Population above the 95th percentile, 10% above the 90th percentile, etc. It will be seen from Table 5.3 that this is not the case. Countries with a higher rate of retention bring less than five percent to the 95th percentile. Although in general the less the intake the better the performance, there are some interesting differences among countries with similar enrolments. Scotland, England, France, Netherlands, Germany, and Belgium all have similar sizes of intake, but differ considerably in the proportions of the enrolment they bring to the international top three percentile levels.

Although the suggestion that "more means worse" has been seen to have some justification, in particular in the bottom half of the distribution, it is more meaningful to see whether the size of the "élite" group (as a proportion of the total age group) can be increased by increasing the size of the intake. If the numbers reaching particular percentile levels are calculated as percentages of the *whole age group*, some differences may become apparent. These percentages are presented in Table 5.4.

The rank order correlations between the percentage of an age group enrolled in the mathematics-science programme and the percentage of the whole age group reaching various percentile levels are given in the last row of Table 5.4.

Table 5.4. *Percentage of age group reaching given standards.*
(Population 3a)

Country	Retentivity	International percentiles					
		25th	50th	75th	85th	90th	95th
	(1)	(2)	(3)	(4)	(5)	(6)	(7)
U.S.A.	18	6.5	3.2	1.6	1.3	.81	.65
Sweden	16	13.0	8.5	4.2	2.1	1.28	.50
Australia	14	9.4	5.2	1.4	.7	.42	.15
Japan	8	6.6	5.0	3.4	2.3	1.68	.80
Scotland	5.4	4.5	2.4	.8	.5	.32	.19
Finland	7	5.7	3.4	1.3	.4	.24	.08
England	5	4.7	3.9	2.5	1.7	1.30	.60
France	5	4.6	3.4	1.9	1.5	1.10	.45
Netherlands	5	4.8	3.8	1.7	.7	.25	.06
Fed. Rep. of Germany	4.7	4.2	3.0	1.2	.5	.32	.09
Belgium	4	3.6	2.8	1.8	1.2	.92	.84
Range		9.4	6.1	3.4	1.9	1.44	.78
Rank correlation with column 1		+.89	+.55	+.15	+.25	+.14	+.10

These positive correlations indicate that the higher the enrolment is as a percentage of the *total age group*, then the higher is the percentage of the whole age group reaching various international percentile levels. The greatest changes from Table 5.3 to Table 5.4 occur in Sweden, the U.S.A. and Japan, all three countries with a more retentive system at the secondary level. Thus, it is possible to increase the size of the élite group (as a percentage of the total age group) but only to a small extent.

Again, the between-country range varies from 9.4 percent at the 25th percentile to .78 percent at the 95th percentile. The percentage of the whole group reaching particular international percentile levels is obviously a function of size of enrolment to a large degree at lower levels though less so at the top levels. It is perhaps not without significance that students reaching the 99th percentile are drawn only from the U.S.A., Sweden, and England (.18, .16, and .05 respectively of their respective total age groups).

Performance of the élite group (in terms of the top ten and five percent international group), is weakly associated with size of enrolment. It is Japan, Sweden, England and Belgium which are performing well. Perhaps the significance of this finding becomes more apparent when phrased in another way: it would appear that countries with higher retentivity are capable of bringing their best pupils (in terms of the same percentage of a year group) to the same standards as less retentive (more selective) countries, i.e., higher retentivity does not necessarily mean lowering the standards of achievement (at least in mathematics) of the better students.

Similar information for Population 3b is given in Tables 5.5 and 5.6. The results agree closely with those obtained for Population 3a. There is a negative relationship (except at the 95th percentile) between the percentage still at school and the percentage of that population reaching various international percentile levels. The small size of the negative correlations for the 75th, 85th and 90th percentiles and the positive correlation at the 95th percentile indicate that at these levels, the degree of retentivity is irrelevant or, at the top level, favourable for high scores. Again, as with 3a, if the numbers reaching the various percentiles are calculated as proportions of the *total age group*, there are positive correlations.

Retentivity in the terminal mathematics-science programme is *negatively* related to the proportions of those still at school reaching

Table 5.5. *Percentage of pre-university non-mathematics students reaching given standards.* (Population 3b)

Country	Retentivity	International percentiles 25th	50th	75th	85th	90th	95th
	(1)	(2)	(3)	(4)	(5)	(6)	(7)
U.S.A.	52	30	12	3	2	1	1
Japan	49	81	60	38	28	21	12
Scotland	12.6	82	50	18	7	3	1
Belgium	9	93	63	27	15	8	2
England	7	84	53	20	10	5	2
Sweden	7	56	10	2	0	0	0
Finland	7	90	57	17	10	5	1
Fed. Rep. of Germany	6.5	99	81	37	20	8	1
Range		69	71	36	28	21	12
Rank correlation with column 1		−.99	−.28	−.02	−.09	−.06	+.38

various international percentile levels. Retentivity is *positively* related to the proportion of the *total* age group reaching various international levels. In general, the systems having smaller intakes of either 3a or 3b have achieved a fairly high performance of the weaker students in the programme. When an intake is increased in size, it is the performance of this lower group which tends to deteriorate. Nations can, however, certainly increase their total "mathematical yield" of an age group by having larger intakes (higher retentivity). In terms of the top international ten and five percents, retentivity is only weakly related to the proportions of the total age group reaching these levels, i.e., the performance of high ability students is unlikely to be affected by increasing the intake.

In Population 3a, Belgium, England and Japan have a consistently high performance of all students. Sweden and Japan demonstrate very well that increasing the size of the intake does not necessarily mean lowering standards. Sweden has an intake approximately three times larger than, for example, that of England, and yet approximately the same proportions of the total age group are still reaching 90th and 95th percentiles. Again, although systems with smaller intakes bring these students to higher mean scores, this is only to be

Table 5.6. *Percentage of age group reaching given standards.* (Population 3b)

Country	Retentivity	International percentiles 25th	50th	75th	85th	90th	95th
	(1)	(2)	(3)	(4)	(5)	(6)	(7)
U.S.A.	52	15.2	6.2	1.6	1.0	.52	.52
Japan	49	39.7	29.4	18.6	13.7	10.3	5.9
Scotland	12.6	10.3	6.3	2.3	.88	.38	.13
Belgium	9	8.4	5.7	2.4	1.3	.72	.18
England	7	5.9	3.7	1.4	.70	.35	.14
Sweden	7	3.9	.70	.14	0	0	0
Finland	7	6.3	3.9	1.2	.70	.32	.07
Fed. Rep. of Germany	6.5	6.4	5.3	2.4	1.3	.52	.06
Range		35.8	28.7	18.6	13.7	10.3	5.9
Rank correlation with column 1		.81	.95	.34	.40	.53	.81

expected when the selection processes and smaller numbers are considered. What is more important, however, is the proportion of the total age group reaching particular levels. Here the size of intake may have an important effect at the lower levels (see Table 5.4, Sweden at 25% level), and at the top levels it is possible for countries with large intakes (e.g. the United States and Sweden) to bring high proportions of an age group to the 90th and 95th international percentiles. At the top level Finland, Australia, the Netherlands and Germany are performing extremely poorly. Germany is particularly surprising, considering its high selectivity. From Table 5.3 it appears that the weaker half of the U.S.A. group is below the standards of other countries.

For Population 3b, Japan, Belgium and Germany perform well, whereas Sweden and the United States perform relatively poorly. It must be remembered that in Germany the 3b group have all studied mathematics up to the end of the penultimate preuniversity year (i.e. the *Unterprima*).

It is interesting to note those countries whose Populations 3a and 3b both perform well and those where there is considerable disparity. However, before arriving at any firm conclusions, it is necessary to bear several points in mind. First, there are differences between systems as to when students are allowed to discontinue the study of mathematics. Secondly, there are differences as to what discontinuing means; in some countries, it means absolutely no further mathematics and in other countries it means having mathematics for one or two periods a week instead of seven or eight periods a week. Thirdly, it must be borne in mind that the distinction between Populations 3a and 3b is somewhat circular, since, where it was difficult in some countries to distinguish between those pre-university students who were said to be specialising in mathematics and those who were not, a way of operationalising the distinction was to give the 3a tests to those groups of students for whom the tests were thought to be appropriate and then the 3b tests to the rest of the students.

Another approach to this same problem described in Husén *et al.* (1967) was to compare the performances of equal proportions of an age group; as a result, the same conclusion as above was reached, i.e., that the performance of the best three or four percent of students in a country is not affected by an increase in the intake (retentivity) into the pre-university year, but that the *average* score of all those in school in either the mathematics or non-mathematics programme will fall as the proportion of an age group retained increases.

Yield

As has already been pointed out in examining the "outcomes" of a system of education, it is often misleading to compare mean scores. It would be pointless to compare the mean score of the English students in the mathematics programme in the pre-university year with the United States students in the 12th grade mathematics programme. It is imperative to take into account the proportion of an age group still studying mathematics, i.e., *"how many of these students are brought how far?"* For example, in England only five percent of an age group is studying mathematics in the pre-university year, whereas in the United States eighteen percent of an age group is studying mathematics at that point.

There are difficulties connected with the calculation of a "yield" or "output" measure. A simple statement of the overall problem is "How are achievement scores and number of students having a given score to be combined into a single measure of output?" Two very simple approaches are used here. The first consists of plotting the cumulative percentile frequencies (or percentile frequencies could be used) against the percentage of an age group in a particular target population and regarding the area under the curve as the "yield". The second consists of multiplying the proportion of an age group in a target population by the mean score of the population and regarding the resultant value as an index of "yield".

The difficulties with these approaches are best exposed by considering the assumptions behind them:

(a) Each correct response to an item is regarded as being of equal value. Thus, two students having the same scores are regarded as representing the same output even though one student may have correct responses on items which are considered to be either more difficult or of more value to society than another student.
(b) Each point on the achievement scale has the same absolute value as every other point. Thus, the increment from 23 to 24 represents the same increase in "output" or "yield" as an increase from 40 to 41. It is, of course, possible that, in some case, 20 points may be twice as valuable as 10, and, in another case, 40 may be less than twice as valuable as 20.
(c) One student with a score of 20 is considered equal in terms of yield to two students with scores of 10 each.
(d) The value of the *n*th unit of achievement is assumed to be the same in all countries, although countries may differ in their economic structure. This, however, introduces the concept of "required (by the society) yield" and its fit to "acquired yield".

Despite the problems involved in calculating "yield", the simple approaches mentioned above will be presented since the concept of "yield" or "output" is important. As has already been mentioned in Chapter 1, what is reported here are the yields of specific target populations. To obtain a measure of the "total yield" of a school system, the achievement of all those dropping out of school has to be measured as they drop out and in some way brought into a single measure. A longitudinal approach could also be adopted.

The yield of students in Population 3a will be examined first, followed by that of the total pre-university year (Populations 3a and 3b combined) and finally the yield of 13-year-olds will be compared with the pre-university yields in each country.

Population 3a

Figure 5.3 represents the yield diagrammatically by plotting the cumulative percentile frequencies for each country against the proportion of an age group still retained in the terminal mathematics-science programme. These distributions have been smoothed graphically. From Figure 5.3 it can be observed that it is Sweden, the United States, Australia and Japan which have the highest yields, despite the fact that in the first three countries the average scores were relatively low. Obviously, yield is, to a certain extent, a function of retentivity, but *only to a certain extent*. The United States' yield is obviously smaller than those of Sweden and Australia, although the United States' retentivity is higher.

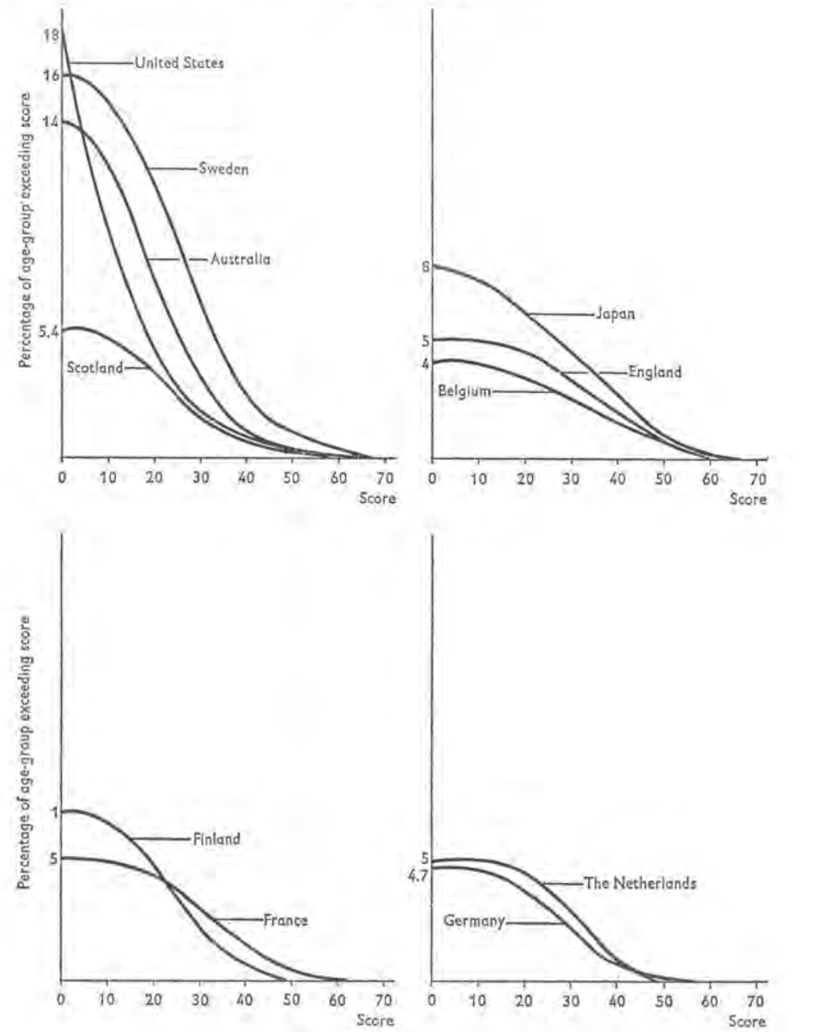
It is interesting to note that in some countries there is a consistently higher performance over the whole range of students than in others (e.g. Japan as compared with Finland). The United States' students at the lower and of the distribution perform less well than the Swedish students. French and English students perform relatively well at the top end of the distribution.

Population 3a and 3b

Although it is only Population 3a which can be regarded as the mathematical "fruits" or "end-products" of a system of education, it is also of interest to examine the yield of Populations 3a and 3b together, since this comprises the total proportion of an age group still in *full*-time schooling. What the yield would be of a total age group is a matter of pure speculation, since in this study no effort was made to measure the mathematics performance of those students in part-time education (and here the proportions of an age group in part-time schooling, whether compulsory or voluntary, differ considerably from country to country) or those young people of the age group not receiving any form of schooling. For example, in England there is a small proportion of an age group which studies pre-university mathematics at Colleges of Further Education or Technical Colleges, but such students were excluded from the target population. In the Federal Republic of Germany a considerable proportion of young people attend *Berufschulen* and continue the study of mathematics there. Again, these students were excluded from the target population, since they were not in *full*-time schooling. Thus, the

Fig. 5.3. *Cumulative Percentile Frequencies (Smoothed)*

(Population 3 a)



"yield" examined here is simply that of all pre-university students in the target populations.

Table 5.7 presents for each country the corrected mean score for Populations 3a, 3b and 1b along with the proportion of the age group still retained in school for each of these populations.

Table 5.7. *Total mathematics score and proportion of age group in school.* (Populations 1b, 3a and 3b)

Country	Population 1b		Population 3a		Population 3b	
	Mean	Proportion	Mean	Proportion	Mean	Proportion
Belgium	30.4	100	34.6	4	24.2	9
England	23.8	100	35.2	5	21.4	7
Fed. Rep. of Germany	25.4	100	28.8	4.7	27.7	6.5
Finland*	16.1	100	25.3	7	22.5	7
Japan	31.2	100	31.4	8	25.3	49
Scotland	22.3	100	25.5	5.4	20.7	12.6
Sweden	15.3	100	27.3	16	12.6	7
U.S.A.	17.8	100	13.8	18	8.3	52

* Although the mean for Finland is given as 16.1 the *scaled* means (and yields) were calculated on uncorrected Finnish data where the mean was 26.4.

However, since Test 5 was common to both Populations 3a and 3b it was possible to estimate[3] what the 3a students would have scored on the 3b tests had they performed in the same way as they did on Test 5. Furthermore, since Test 3 was common to Populations 3b and 1b, it was possible to estimate what 1b students would have scored on the 3b tests had they performed in the same way as they did on Test 3.[3] However, it must be remembered that the content of 3a and 3b tests differed considerably from Test 5 and also the content of the 3b and 1b tests from Test 3, as can be seen in the Appendix to Volume II of Husén *et al.* (1967); this accounts for the differences between Table 5.7 and Figure 5.4, where scaled means are given.

[3] A regression procedure was used for each country to predict a test 3 score (t_3) from the total level 1b score (T_{1b}) and then predicting from that t_3 to an estimated T_{3b} on the 3b scale. The two regression equations were:

$$t_3 = a_1 + b_1 T_{1b}$$

and

$$\hat{T}_{3b} = a_3 + b_3 t_3$$

which combine to give

$$\hat{T}_{3b} = a_3 + a_1 b_3 + b_1 b_3 T_{1b}$$

where

$$b_1 = \frac{N\Sigma T_{1b} t_3 - (\Sigma t_3)(\Sigma T_{1b})}{N\Sigma T_{1b}^2 - (\Sigma T_{1b})^2}$$

$$a_1 = \bar{t}_3 - b_1 \bar{T}_{1b}$$

$$b_3 = \frac{N\Sigma t_3 T_{3b} - (\Sigma t_3)(\Sigma T_{3b})}{N\Sigma t_3^2 - (\Sigma t_3)^2}$$

$$a_3 = \bar{T}_{3b} - b_3 \bar{t}_3$$

The same procedure was used for reducing the 3a to 3b score.

Table 5.8. *Correlations of tests 5 and 3 with total mathematics score.*

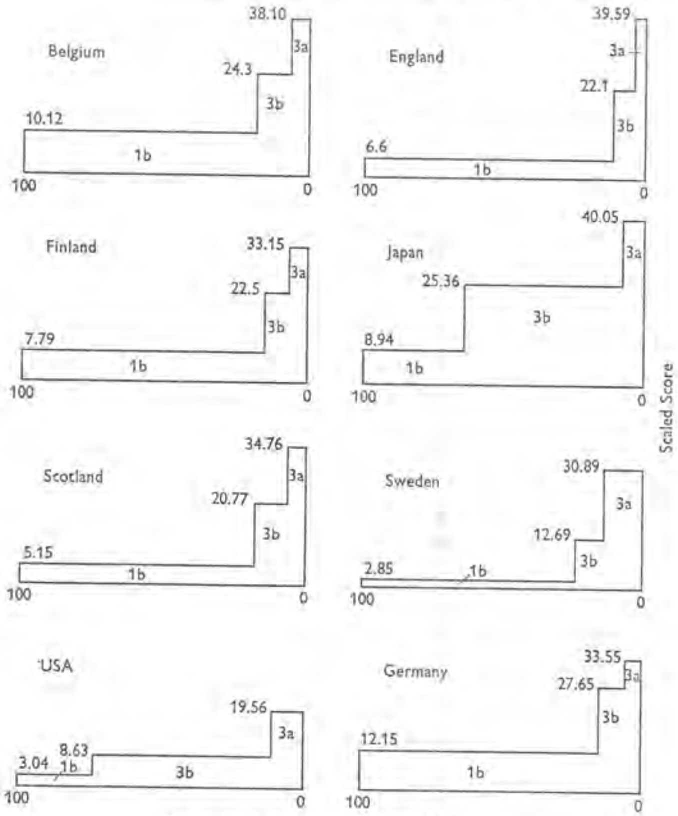
Country	Population 3a	Population 3b		Population 1b
	Test 5	Test 5	Test 3	Test 3
U.S.A.	.91	.86	.90	.79
Japan	.90	.94	.91	.90
Sweden	.86	.73	.80	.78
Scotland	.82	.87	.88	.87
Finland*	.84	.85	.84	.78
Belgium	.86	.86	.85	.85
England	.88	.87	.88	.90
Fed. Rep. of Germany	.78	.80	.79	.82
All Countries	.89	.92	.91	.86

* These correlations were calculated on the uncorrected Finnish data.

Table 5.8 presents the product moment correlation coefficients between the Total Mathematics Scores (corrected) and Test 5 and Test 3 scores (as the case may be) for each population in each country. The Total Mathematics Score included the Test 5 (or 3) scores and hence the correlations are higher than if it were Test 5 (or 3) scores correlated with the Total Mathematics Score minus Test 5 (or 3).

Figure 5.4 presents the diagrams of scaled means for Populations 3a, 3b and 1b against the proportion of an age group still in school for each of these populations. Each diagram is made up of three parts as follows. The base of each diagram consists of the 1b population (where 100 percent of an age group is estimated to be in full-time schooling); the proportion of an age group is shown on the horizontal axis and the scaled mean score on the vertical axis. A similar procedure is used for the 3b population and for the 3a population shown at the right side of each diagram.

Fig. 5.4. *Combined Yield (3a + 3b on 1b)*
(Regression Scaling of 3a + 1b on to 3b)



Proportion of Age Group in Target Populations

In Figure 5.4 the effect of retentivity on yield can be seen. Japan has a particularly large yield. It should, however, be remembered that the procedure used here does not take into account those students who have left school between Populations 1 and 3.

It is possible to calculate a yield coefficient for each population by multiplying the scaled mean (or ordinary mean) by the percentage of an age group in school. The percentage of an age group in school for the 1b population is estimated to be 100% in each country. The combined yield of the pre-university year is the sum of the yield coefficients for Populations 3a and 3b. These yield coefficients are given in Table 5.9 (the scaled means are given in Table A.4 in the Appendix).

Table 5.9. *Yield coefficients.*

	On scaled means				On ordinary means			
Country	1b	3a	3b	3a + 3b	1b	3a	3b	3a + 3b
U.S.A.	304	352	449	801	178	2484	4316	6800
Japan	894	320	1243	1563	312	2512	12397	14909
Sweden	285	494	89	583	153	4368	882	5250
Scotland	515	188	263	451	223	1377	2608	3985
Finland	779*	232	157	389	161	1771	1575	3346
Belgium	1012	152	219	371	304	1384	2178	3562
England	660	198	155	353	238	1760	1498	3258
Fed. Rep. of Germany	1215	158	180	338	254	1354	1800	3154

* The scaled means were calculated on the uncorrected Finnish data. It has not been possible to rerun the regression scaling analyses since the mistake in the Finnish data was discovered.

The rank correlations between the scaled mean yield coefficients and the ordinary mean yield coefficients are .79, .91, 1.0 and .98 for 1b, 3a, 3b and 3a plus 3b respectively. The correlations indicate that there is a high degree of relationship between the two types of means used to calculate the yield coefficients. In terms of the pre-university yield (3a + 3b) it is worthy of note that although the United States has three times as many pupils as Sweden enrolled in the pre-university year, its yield is only 25 percent greater. Again, Japan has just over twice as many pupils as Sweden enrolled in the pre-university year, but has a yield nearly three times as great.

It is of particular interest, when considering yield, to compare the yield of the 1b population with the pre-university yields. Since the 13-year-old grade group was the last point in all the school systems where 100 percent of an age group was still in school, it can be considered as a comparable point near to the end of compulsory schooling, and the yields as fairly representative of the outcomes of the compulsory schooling in each country. At the same time, it must be realised that the actual age of ending compulsory schooling differs from system to system and that some countries will obviously increase this yield before the end of compulsory schooling.

It seems likely, for example, that, in those countries where compulsory schooling does not end until the age of sixteen, certain topics which are considered to be difficult may be postponed until the age of fifteen, while in those countries where compulsory education finishes at fourteen years of age, these topics may be introduced at thirteen years of age. It might have been better to use Population 1a instead of 1b for these yields, since this is a strictly chronological group, but as pointed out earlier in this chapter, this would have provided results for seven countries only, since Germany did not test Population 1a. Therefore, despite the limitations involved, it was decided to use Population 1b.

The rank correlation between the 1b yields and 3a+3b yields is –0.56. Germany and Belgium are particularly worthy of note here, since from the 1b yield to the 3a+3b yields they move from first and second places to last and 6th respectively. Only the United States, Japan and Sweden have relatively higher yields at the pre-university level than at the 1b level and this is obviously, to a certain extent, a function of the size of retentivity. It would seem that the less retentive systems lose a great deal of potential mathematical knowledge in their countries, and, at the same time must also lose a certain amount of talent. The rank correlation between yields (scaled) for Populations 3a and 3b (separately) and the measures of social bias[8] given in Chapter 3 of Volume II of Husén *et al.* (1967) are +.56 and +.56. (The measures of social bias are repeated in Table A.5 in the Appendix to this book).

Thus it can be seen that the pre-university yield is negatively related to the 1b yield, but is positively related to social bias which is in turn related to the age at which selection takes place (see Husén *et al.*, 1967). At the same time, we know that yield is, to a certain extent, a function of retentivity and retentivity is related to the percentage of pupils in Population 1a in comprehensive schools (see p. 67). It would seem that in countries with higher yields at the pre-university level, there is a philosophy of equality of opportunity in that selection is delayed or abolished, comprehensive schools are more common and more pupils from lower social status families continue through to the end of secondary schooling.

These organizational features, however, are not alone responsible for high yields, as seen by the difference between the United States', Japanese and Swedish yields. The curriculum, teaching and other

[8] Social bias is an index of the degree of difference of the socio-economic composition of one group to another, in this case Population 1a to 3a and 1a to 3b. It can be reasonably assumed that Populations 1a and 1b have nearly identical socio-economic distributions.

family background characteristics are the most likely factors to account for other differences (see Chapter 6 of Volume II in Husén *et al.*, 1967).

Although some factors associated with yield have been examined, no mention of the relationship between this yield "acquired" by the systems and the yield "required" by a society has been made, since it is not known. Research similar to that carried out by Dahllöf (1963) would have to be undertaken where different branches of society receiving students from school could estimate the amount of knowledge they require from these students in a particular subject, and where, at the same time, approximations could be made of the proportion of any one age group entering work in that branch of society. In this way, it would be possible to estimate the "required" yield. Yield, as discussed in this chapter, has been based on Total Mathematics Score; it would, of course, be possible to discuss yield in terms of particular topics in mathematics and clusters of topics. By comparing "required" with "acquired" yield, it would be possible to examine how well the schools prepare their students to meet the needs of the society. This is *not* to imply that a school system should be based on a purely utilitarian philosophy; it should, of course, have much wider aims. Nevertheless, one of its basic tasks should be to meet the needs of the society. At present, however, the only system, to the author's knowledge, where this problem of "required" yield has begun to be examined empirically is Sweden. In other countries, there is only intuitive knowledge of what society requires.

Although it is possible to obtain ratings of the amount and type of mathematical knowledge required by various sectors of the society (including the university) receiving students straight from school, the problem becomes difficult when prediction in terms of manpower requirements with certain mathematical competences is attempted —the concept of "fit". This is so because, in the economist's language, "demand" is never a fixed amount but rather a schedule. Furthermore, the principle of substitution operates so that to some extent x "poorer" mathematicians can be substituted for y "better" mathematicians. Thus, the question becomes that of how many mathematicians are desired at each alternative price per unit. Added to this is the problem of predicting future demands. What is self-evident is that in the application of the concept of "fit" an interdisciplinary attack is required.

Summary

It has been shown, by a discussion of the relationship between retentivity and type of school system, how the traditional European system, involving selection into an academic secondary school, has a lower rate of retentivity than the United States' system with its self-contained establishments which have been continually expanded to satisfy the educational needs of the community. However, in many of the European countries at present, policies concerning school structure are being revised, and in Sweden, for example, the percentage of seventeen-year-olds proceeding to *gymnasiet* has risen from ten to 28 percent from 1950 to 1964 (Dahllöf *et al.*, 1966).

The percentage of an age group in both the pre-university mathematics and non-mathematics programmes in the twelve countries in the study was related to an index of comprehensive schooling in the various countries and the correlations obtained were high. Three examinations of the pre-university scores were made in connection with retentivity: an examination of average performance of the various populations, an examination of the performance of the various populations at fixed international standards and an examination of the "yield" (how many are brought how far) of the various populations. Variations among the pre-university populations in such characteristics as age, social class composition and number of subjects studied are pointed out.

The examination of average performance shows that countries which retain larger percentages of an age group to the pre-university stage produce on *average* lower standards of achievement than do countries retaining smaller percentages. However, the range of scores was not related to retentivity, although this would have been expected.

In the examination of performance at various fixed international standards it becomes clear that although the *average* score may drop when a higher proportion of an age group goes through to the pre-university year, the performance of the best students (in terms of the proportion of a year group reaching various international percentile levels) does not necessarily deteriorate. In other words, an increase in intake into the pre-university year does not necessarily cause a drop in the levels of achievement of the best students. This finding is of particular importance in the light of the fears of many teachers who argue that if more and hence poorer students are allowed through, the standards of performance will deteriorate and the learning of the better students will suffer.

Since this is the case, it is interesting to proceed to an examination of the "yields" (how many are brought how far) in mathematics of the pre-university populations in the eight countries which had scores for both Populations 3a and 3b. "Yield" takes into account the differing proportions of an age group in these populations in the different countries, whereas a comparison of *average* performances of pre-university year students in different countries does not. A diagrammatic presentation of "yield" for Population 3a is given, and this is also given in terms of "yield coefficients" (calculated on scaled mean scores as well as ordinary mean scores) for both Populations 3a and 3b. In general, systems with higher retentivity have greater yields, but yield is, to a certain extent, a function of retentivity. Curriculum, student motivation and other factors also would seem to play some part in accounting for other differences in performance. It would seem that further research is needed to explore these issues. The relationship between Population 1b yields and the pre-university yields was negative and is mainly, but not entirely, due to the varying retentivity through to the pre-university year. It would seem that in some countries, particularly Germany and Belgium, a great deal of talent drops out of regular full-time schooling. This is, in turn, related to the selection process in some countries and results in bias in the social status composition of the students in the pre-university years in favour of the higher social status groups. The data obtained in this study reveal clearly the possibility of having both a high overall yield and an undiminished élite yield.

Although the concept of "yield" or "output" introduced is somewhat crude, it is an important one and it is to be hoped that its conceptualisation and operationalisation will be pursued, and that it can be so refined in the future to produce detailed measures of "acquired" yield in many subject areas. Measurement of "required" yield has already been begun in some areas. When progress is made in the measurement of the types of yield—that produced by the school system and that required by society—it will be possible to compare them and although the concept of "required yield" has its difficulties, the whole notion of "fit" may provide the schools (and educational policy makers) with more insight into the ways and means of catering for the needs of society.